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On the Equivalence of Different Formulations of the M Theory Five–Brane

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ABSTRACT

We show that the field equations for the supercoordinates and the self–dual antisymmetric tensor field derived from the recently constructed κ–invariant action for the M theory five-brane are equivalent to the equations of motion obtained in the doubly supersymmetric geometrical approach at the worldvolume component level.

A super–five–brane sigma model in eleven dimensions is an essential ingredient of M–theory, an eleven–dimensional theory that has been conjectured in the framework of string duality.

This brane contains on its worldvolume a self–dual tensor whose presence hampered for a long time a complete description of this extended object due to known problems with (manifest) Lorentz invariance. Only very recently consistent worldvolume formulations of the super–five–brane in $D = 11$ have been proposed.

A complete covariant Dirac–Born–Infeld (DBI)–like action for the bosonic $D = 11$ five–brane was constructed in [1], and in [2,3] it was generalized to a k–invariant action for the M–theory super–five–brane in $D = 11$ supergravity backgrounds. The construction is based on a previous knowledge about the structure of different parts of the action obtained in [4,5]. A manifestly Lorentz invariant treatment of the self–dual field was achieved by applying a method proposed in [6]. The method consists in introducing an auxiliary scalar field which does not propagate and can be eliminated by fixing a new local symmetry of the action at the expense of losing manifest Lorentz invariance. The five–brane action in this gauge was considered in detail in [3]. An advantage of having the covariant formulation is that the five–brane action has a conventional worldvolume diffeomorphism invariant form, which simplifies the analysis of its structure and relation with other extended objects, for instance, with a heterotic string [7].

The presence of the auxiliary field also reveals nontrivial topological properties of the model.

At first sight a completely different approach is that of Refs. [8,9]. It is based on a doubly supersymmetric geometrical approach to describing super–p–brane dynamics [10–14] (and refs. therein), where both the five–brane worldvolume and the eleven–dimensional target space are superspaces and what one gets are geometrical conditions specifying a superembedding of worldvolume superspace into a target superspace. In many cases, as it happens with the five–brane, these conditions put the theory on the mass shell and yield superfield equations of motion without any knowledge on the structure of the five–brane action. Thus, this method does not furnish any action from which these equations can be derived.

In this letter we shall prove that the approaches of [1,2,3] and that of [8,9] are equivalent in the sense that the worldvolume component equations of motion (for X, ϑ and the chiral tensor) in the latter are equivalent to the equations of motion derived from the action in the former. This result is not so surprising, since, after a double dimensional reduction [1,9,15], both formulations yield a DBI–like structure of the D–four–brane in $D = 10$ and, moreover, as shown in [16], the field equations of the chiral tensor are the same in the both cases. However, in Ref. [9] the authors presented κ–variations of their component fields which look rather different from those in the action

approach. Here we will show that the two sets of κ–transformations are in fact related by a redefinition of the parameter κ.

The paper is organized as follows. We start with a review of the structure of the five–brane action, then consider the five–brane component equations obtained by use of the geometrical approach, and finally derive a set of relations and identities which establish the equivalence between the two formulations.

The five–brane is described by the supercoordinates

$$Z^{\underline{M}}(x) \equiv (X^{\underline{m}}, \vartheta^{\underline{\mu}}) \ \ (\underline{m} = 0, ..., 10; \ \underline{\mu} = 1, ..., 32).$$

together with the worldvolume $2-$form $A_2(x)$ which is the potential of a self–dual tensor. The x^m are the coordinates of the brane worldvolume (underlined indices refer to the eleven–dimensional target superspace while indices which are not underlined refer to the six–dimensional worldvolume). The curved target superspace background is specified by the supervielbeins $E^{\underline{A}}(Z) \equiv dZ^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}(Z)$ and the Lorentz superconnection $\Omega_{\underline{A}}{}^{\underline{B}}(Z)$ with torsion $T^{\underline{A}} = DE^{\underline{A}}$ and Lorentz curvature $R_{\underline{A}}{}^{\underline{B}}(Z)$, together with the target space three–superform $C_3(Z)$ and six–superform $C_6(Z)$ with curvatures

$$R_4 = dC_3 \tag{1}$$

$$R_7 = dC_6 + \frac{1}{2} C_3 \wedge R_4. \tag{2}$$

C_3 and C_6 are dual potentials in the sense that

$$R_{\underline{a}_1 \cdots \underline{a}_7} = \frac{1}{4!} \epsilon_{\underline{a}_1 \cdots \underline{a}_7}{}^{\underline{b}_1 \cdots \underline{b}_4} R_{\underline{b}_1 \cdots \underline{b}_4}. \tag{3}$$

The torsion and the curvatures are restricted by suitable constraints describing on–shell $D = 11$ supergravity. In particular, (see, for instance, [17])

$$T_{\underline{\alpha\beta}}{}^{\underline{a}} = 2\Gamma^{\underline{a}}_{\underline{\alpha\beta}} \tag{4a}$$

$$R_{\underline{ab\alpha\beta}} = -2i(\Gamma_{\underline{ab}})_{\underline{\alpha\beta}} \tag{4b}$$

$$R_{\underline{a}_1 \cdots \underline{a}_5 \underline{\alpha\beta}} = -2(\Gamma_{\underline{a}_1 \cdots \underline{a}_5})_{\underline{\alpha\beta}}. \tag{4c}$$

(In fact, one should not impose these constraints *a priori*, since they arise as consistency conditions for the five–brane action to be κ–invariant.)

For the worldvolume two–form A_2 we define the gauge invariant field strength

$$H_3 = dA_2 + C_3 \tag{5}$$

so that

$$dH_3 = R_4. \tag{6}$$

In eqs. (5) and (6) we imply the pullback of C_3 onto the worldvolume. This pullback will always be understood in what follows. Our convention for target superforms is

$$\psi_N = \frac{1}{N!} E^{\underline{A}_1} .. E^{\underline{A}_N} \psi_{\underline{A}_N \dots \underline{A}_1}$$

and those in the worldvolume

$$\Phi_n = \frac{1}{n!} dx^{m_1} ... dx^{m_n} \Phi_{m_n \dots m_1}.$$

Latin and greek letters denote respectively vector–like and spinor–like indices, those from the beginning of the alphabet denote target space indices.

The five–brane action proposed in [1,2] contains the fields Z^M, A_2, and a scalar worldvolume auxiliary field $a(x)$, which insures $d = 6$ covariance of the construction:

$$I\left[Z, A, a\right] = \int d^6x \sqrt{-g} \left(\mathcal{L} + \frac{1}{4} \tilde{H}^{mn} H_{mn} \right) - \int \left(C_6 - \frac{1}{2} C_3 \wedge H_3 \right). \qquad (7)$$

Here

$$g_{mn}(Z) = E_m{}^{\underline{a}}(Z) E_n{}^{\underline{b}}(Z) \eta_{\underline{ab}} \qquad (8)$$

is the induced worldvolume metric, which we use to raise and lower (curved) six–dimensional indices, and $E_m{}^{\underline{A}} = \partial_m Z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}$. $\mathcal{L}$ is the DBI–like Lagrangian

$$\mathcal{L} = \sqrt{det(\delta_m{}^n + i\tilde{H}_m{}^n)}. \qquad (9)$$

The auxiliary field $a(x)$ enters the action under derivative. It is convenient to define its vector "field strength" as

$$v_m = \frac{\partial_m a}{\sqrt{-g^{pq} \partial_p a \partial_q a}} \quad , \qquad (10)$$

then in (8) H_{mn} and $\tilde{H}^{mn}$ are defined as follows

$$H_{mn} = H_{mnl} v^l, \qquad (11)$$

$$\tilde{H}^{mn} = H^{*mnl} v_l. \qquad (12)$$

Note that $v_m v^m = -1$, which allows one to manipulate with v_m as with a sechsbein component and to essentially simplify many computations. The dual of H is defined in the standard way as

$$H^{*mnl} = \frac{1}{3! \sqrt{-g}} \epsilon^{mnlpqr} H_{pqr},$$

and one has the identical decomposition

$$H^{mnl} = -\frac{1}{2} \frac{\epsilon^{mnlpqr}}{\sqrt{-g}} v_p \tilde{H}_{qr} - 3 v^{[m} H^{nl]} \qquad (12a)$$

together with the analogous formula for H^*.

The action (7) is invariant under the local symmetries

$$\delta_1 Z^{\underline{M}} = 0, \qquad \delta_1 a = \varphi(x), \qquad \delta_1 A_{mn} = \frac{-\varphi}{2\sqrt{-g^{pq}\partial_p a \partial_q a}} \left(H_{mn} - \mathcal{V}_{mn}\right), \tag{13}$$

where

$$\mathcal{V}_{mn} \equiv -2\frac{\delta \mathcal{L}}{\delta \tilde{H}^{mn}}, \tag{14}$$

and

$$\delta_2 Z^{\underline{M}} = 0 = \delta_2 a, \qquad \delta_2 A_{mn} = \partial_{[m} a \quad \Phi_{n]}, \tag{15}$$

where $\varphi(x)$ and $\Phi_n(x)$ are infinitesimal transformation parameters, as well as under the standard gauge symmetry of $A_2(x)$, the $d=6$ diffeomorphisms and κ–symmetry. The latter is defined by

$$\delta_\kappa Z^M E_M{}^{\underline{\alpha}} = \Delta^{\underline{\alpha}}, \qquad \delta_\kappa Z^M E_M{}^{\underline{a}} = 0, \qquad \delta_\kappa A_2 = -i_\Delta C_3 \qquad \delta_\kappa a = 0. \tag{16}$$

Here

$$\Delta^{\underline{\alpha}} \equiv \left(1+\bar{\Gamma}\right)^{\underline{\alpha}}{}_{\underline{\beta}}\, \kappa^{\underline{\beta}}, \tag{17}$$

$$\bar{\Gamma} = \frac{1}{\mathcal{L}}\left(\bar{\gamma} + \frac{i}{2}\Gamma^{mnp} v_m \tilde{H}_{np} - \frac{1}{16}\frac{\epsilon^{m_1 \ldots m_6}}{\sqrt{-g}} \tilde{H}_{m_1 m_2} \tilde{H}_{m_3 m_4} \Gamma_{m_5 m_6}\right) \tag{18}$$

and

$$\Gamma_m = E_m{}^{\underline{a}} \Gamma_{\underline{a}}, \qquad \bar{\gamma} = \frac{1}{6!\sqrt{-g}} \epsilon^{m_1 \ldots m_6} \Gamma_{m_1 \ldots m_6}. \tag{19}$$

The matrix $\bar{\Gamma}$ satisfies the conditions

$$\bar{\Gamma}^2 = 1, \qquad tr\bar{\Gamma} = 0. \tag{20}$$

Under the κ–transformations the action (7) varies as

$$\delta I = -\int d^6 x \sqrt{-g} E_m{}^{\underline{\beta}} (J^m)_{\underline{\beta}\underline{\alpha}} \Delta^{\underline{\alpha}} \tag{21}$$

where the matrices J^m are

$$J^m = T^{mn}\Gamma_n + 2\Gamma^m \bar{\gamma} + i T^{mnp}\Gamma_{np}, \tag{22}$$

where T^{mn} is the *formal* energy–momentum tensor with respect to the induced metric (it is not conserved)

$$T^{mn} = \frac{-4}{\sqrt{-g}}\frac{\delta I}{\delta g_{mn}} = -2g^{mn}\left(\mathcal{L} - \frac{1}{2}tr(\mathcal{V}\tilde{H})\right) + v^m v^n tr(\mathcal{V}\tilde{H})$$

$$-2(\mathcal{V}\tilde{H})^{mn} - \frac{1}{2}\frac{v^{(m}\epsilon^{n)p_1..p_5}}{\sqrt{-g}} v_{p_1}\tilde{H}_{p_2p_3}\tilde{H}_{p_4p_5} \tag{23}$$

and

$$T^{mnp} = 3v^{[m}\tilde{H}^{np]} + \frac{\epsilon^{mnpqrl}}{2\sqrt{-g}} v_q \mathcal{V}_{rl}. \tag{24}$$

Due to the matrix identity

$$J^m(1+\bar{\Gamma}) = 0 \tag{25}$$

the action is indeed κ–invariant.

The equations of motion of $\vartheta^{\underline{\mu}}$, $X^{\underline{m}}$, A_2 and $a(x)$ are, respectively,

$$\begin{aligned}
E_m{}^{\underline{\beta}}(J^m)_{\underline{\beta\alpha}} &= 0, \\
\frac{1}{2}D_m\left(T^{mn}E_n{}^{\underline{a}}\right) &= \frac{\epsilon^{m_1\cdots m_6}}{\sqrt{-g}}\left(\frac{1}{6!}R^{\underline{a}}{}_{m_6\cdots m_1} - \frac{1}{(3!)^2}R^{\underline{a}}{}_{m_6m_5m_4}H_{m_3m_2m_1}\right), \\
\partial_{[m}\left(v_n\left(\mathcal{V}_{kl]} - H_{kl]}\right)\right) &= 0, \\
\epsilon^{pqmnkl}\left(\mathcal{V}_{pq} - H_{pq}\right)\partial_m\left(v_n\left(\mathcal{V}_{kl} - H_{kl}\right)\right) &= 0.
\end{aligned} \tag{25a}$$

Note that the equation of motion of $a(x)$ is not independent but is a consequence of the A_2 equation. As has been shown in ref. [1], by appropriately fixing the gauge transformations in (15) the equation of motion of A_2 reduces to the generalized self–duality condition

$$H_{mn} = \mathcal{V}_{mn}. \tag{25b}$$

Using Eq. (25b) and the fact that $\int d^6x\sqrt{-g}\left(\mathcal{L} - \frac{1}{4}tr\left(H\tilde{H}\right)\right)$ is $d=6$ diffeomorphism invariant one can show that T_{mn} satisfies the equation

$$D_mT^{mn} = -\frac{2}{(3!)^2}\frac{\epsilon^{m_1\cdots m_6}}{\sqrt{-g}}R^n{}_{m_6m_5m_4}H_{m_3m_2m_1}.$$

This allows one to rewrite the equation of motion of $X^{\underline{m}}$ as

$$\begin{aligned}
&\frac{1}{2}T^{mn}D_mE_n{}^{\underline{a}} = \\
&\quad\frac{\epsilon^{m_1\cdots m_6}}{\sqrt{-g}}\left(\frac{1}{6!}R^{\underline{a}}{}_{m_6\cdots m_1} - \frac{1}{(3!)^2}\left(R^{\underline{a}}{}_{m_6m_5m_4}H_{m_3m_2m_1} - E_{n\underline{a}}E^n{}_{\underline{b}}R^{\underline{b}}{}_{m_6m_5m_4}H_{m_3m_2m_1}\right)\right).
\end{aligned} \tag{25c}$$

We conclude the presentation of the action approach by noting that when the local transformations (13) and (15) are gauge fixed by the conditions

$$a(x) = x^5 \longrightarrow \partial_m a(x) = \delta^5_m; \qquad A_{5m} = 0, \tag{26}$$

one recovers the formulation of [3]. In this gauge the invariance under worldvolume diffeomorphisms is no longer manifest, but still present in a modified form.

Let us now present the five–brane equations of motion [8,9] which follow from the doubly supersymmetric geometrical approach [13]. In this formulation the worldvolume is a supersurface Σ locally parametrized by the bosonic coordinates x^m, $(m = 0, ..., 5)$ and the fermionic coordinates ϑ^μ, $(\mu = 1, ..., 16)$. The $Z^{\underline{M}}$ are now superfields in Σ, and A_2 as well as the pullbacks of target space superforms become superforms in Σ. The essential ingredient of this approach is the requirement that the embedding of the superworldvolume of the five–brane into a target superspace respects the condition that the pullback of $E^{\underline{a}}$ does not have components along the odd directions of the worldvolume supersurface:

$$E_\alpha{}^{\underline{a}} = 0. \tag{27}$$

This condition appeared first in the twistor–like formulation of superparticles [10] and heterotic strings [11] and it is a characteristic property of all superbranes in the doubly supersymmetric geometrical approach [12,13,14]. In many cases, as that of $N = 2$, $D = 10$ superstrings, $D = 11$ supermembranes [12,13] and D–branes [8,14], this condition is so strong that it forces the model to be on the mass shell. In particular, this is the case of the M theory five–brane [8,9]. The details of this case have been worked out in [8,9] and will not be given here. For our purposes it is sufficient to present the equations of motion for the worldvolume component fields (which are again Z^M and A_2) adapting the conventions used in those references to ours. The correct identification of the worldvolume component fields, which turns out to be a nontrivial thing, has already been made in [16]. The curvature H and the target space fields are again defined as in (1)–(6) and all indices are raised and lowered by the induced metric (8).

The basic auxiliary field appearing in this approach is an antisymmetric self–dual tensor which we convert to a tensor with curved indices

$$h^{mnl} = h^{*mnl} = \frac{1}{3!\sqrt{-g}} \epsilon^{mnlpqr} h_{pqr}.$$

The consistency of Bianchi identities implies that this tensor is tied to H through the relation

$$4h_{pqr} = m^l{}_{[p} H_{qr]l}, \tag{28}$$

where

$$\begin{aligned} m_{lm} &= g_{lm} + 2h_l{}^{pq} h_{mpq} \\ &\equiv g_{lm} + 2k_{lm}. \end{aligned}$$

Upon elimination of the auxiliary field h, Eq. (28) turns out to be the field equation for H. With spinor indices suppressed the equation for $\vartheta^{\underline{\mu}}$ can be written as

$$E^m \left(1 - \gamma^{(3)}\right) \Gamma^n \left(1 - \bar{\gamma}\right) m_{mn} = 0, \tag{29a}$$

where the matrix $\gamma^{(3)}$ is given by

$$\gamma^{(3)} = -\frac{i}{3} h_{lmn} \Gamma^{lmn} = -\frac{i}{3} h_{lmn} \Gamma^{lmn} \left(\frac{1+\bar{\gamma}}{2} \right).$$

Eq. (29a) becomes the equation of motion of $\vartheta^{\underline{\mu}}$ once, by use of (28), one expresses h as a function of H, see below.

In [9] the equation of motion of $X^{\underline{m}}$ is given in the approximation in which one neglects $E_m{}^{\underline{\alpha}}$ (i.e. drops all terms bilinear (and higher) in fermions from the bosonic equations)

$$(m^2)^{mn} D_m E_n{}^{\underline{a}} = \\ -\left(1 - \frac{2}{3} trk^2\right) \frac{\epsilon^{m_1 \cdots m_6}}{\sqrt{-g}} \left(\frac{1}{6!} \hat{R}^{\underline{b}}{}_{m_6 \cdots m_1} - \frac{1}{(3!)^2} \hat{R}^{\underline{b}}{}_{m_6 m_5 m_4} H_{m_3 m_2 m_1} \right) \left(\delta_{\underline{b}}{}^{\underline{a}} - E^l{}_{\underline{b}} E_l{}^{\underline{a}} \right). \tag{29b}$$

We put the hat on the fields R_7 and R_4 to remember that the pullback has been made only with respect to their *bosonic* target indices $\underline{b}$, i.e. with $E_m{}^{\underline{b}}$. These component field equations are covariant under κ–transformations which are completely analogous to (16) (together with a suitable transformation law for h), but with the difference that now

$$\Delta^{\underline{\alpha}} = \left(1 + \hat{\Gamma}\right)^{\underline{\alpha}}{}_{\underline{\beta}} \hat{\kappa}^{\underline{\beta}}, \tag{30}$$

where

$$\hat{\Gamma} = \bar{\gamma} + \gamma^{(3)}. \tag{31}$$

Notice that $\hat{\Gamma}$ also satisfies $\hat{\Gamma}^2 = 1$ and $tr\hat{\Gamma} = 0$.

The connection between the two approaches is established as follows. First, one has to disentangle the implicit equation (28). For this define

$$h_{mn} = h_{mnp} v^p = h^*_{mnp} v^p \tag{32}$$

and use the identity (12a) to project H and h onto their dual and self–dual parts. One gets

$$4h_{mn} = \left(1 - \frac{2}{3} trh^2\right) H_{mn} - \frac{2}{3} \left(2tr(\tilde{H}h) h_{mn} - 4(hh\tilde{H})_{mn} - 8(hhH)_{mn} \right) \tag{33}$$

$$4h_{mn} = \left(1 + \frac{2}{3} trh^2\right) \tilde{H}_{mn} + \frac{2}{3} \left(2tr(Hh) h_{mn} - 4(hhH)_{mn} - 8(hh\tilde{H})_{mn} \right). \tag{34}$$

Despite of a complicated form, these equations can be solved to get h_{mn} and H_{mn} in terms of $\tilde{H}_{mn}$:

$$h_{mn} = \frac{1}{4} \tilde{H}_{mn} + \frac{\frac{1}{2}(tr\tilde{H}^2)\tilde{H}_{mn} - 2\tilde{H}^3_{mn}}{8(\mathcal{L}+1) + 2tr\tilde{H}^2} \tag{35}$$

$$H_{mn} = \frac{1}{\mathcal{L}} \left((1 + tr\tilde{H}^2)\tilde{H}_{mn} - \tilde{H}^3_{mn} \right) = -2 \frac{\delta L}{\delta \tilde{H}^{mn}} = \mathcal{V}_{mn}, \tag{36}$$

where $\mathcal{L}$ is the DBI lagrangian (9). Eq. (37) coincides precisely with (25b), i.e. with the self–duality condition for A_2 in the action approach (see also [16] where this condition was obtained in a $d=6$ covariant form without any use of the auxiliary fields).

The simplest way to obtain Eqs. (35,36) is to use manifest diffeomorphism invariance and choose the flat metric $g_{mn} = \eta_{mn}$. Since each of the antisymmetric matrices h, H and $\tilde{H}$ live in five dimensions, following [5] one can perform a five–dimensional Lorentz rotation such that the only nonvanishing components of h_{mn} are

$$h_{12} = -h_{21} = h_{+}, \quad h_{34} = -h_{43} = h_{-}, \tag{37}$$

and similarly for H_{mn} and $\tilde{H}_{mn}$. Then Eqs. (33), (34) become

$$4h_{\pm} = H_{\pm}\left(1 \mp 4(h_{+}^2 - h_{-}^2)\right) \tag{38a}$$

$$4h_{\pm} = \tilde{H}_{\pm}\left(1 \pm 4(h_{+}^2 - h_{-}^2)\right). \tag{38b}$$

Eliminating $h_{\pm}$ one gets

$$H_{\pm} = \tilde{H}_{\pm}\sqrt{\frac{1-\tilde{H}_{\mp}^2}{1-\tilde{H}_{\pm}^2}} = -\frac{\delta\mathcal{L}}{\delta\tilde{H}_{\pm}} = \mathcal{V}_{\pm}, \tag{39}$$

where

$$\mathcal{L} = \sqrt{(1-\tilde{H}_{+}^2)(1-\tilde{H}_{-}^2)}, \tag{40}$$

which is the DBI–like Lagrangian in this particular basis. In an analogous way one can derive (35).

On what concerns the X-equation we note that using (12a) for h we can write the symmetric matrix k in the form

$$k^{mn} = 4(h^2)^{mn} - g^{mn}trh^2 - 2v^m v^n trh^2 + \frac{v^{(m}\epsilon^{n)p_1..p_5}}{\sqrt{-g}}v_{p_1}\tilde{h}_{p_2p_3}\tilde{h}_{p_4p_5}$$

and use it and (25b) to derive a remarkable relation

$$(m^2)^{mn} = -\frac{1}{2}T^{mn}\left(1-\frac{2}{3}trk^2\right).$$

If one uses this relation and drops the terms containing $E_m{}^{\underline{\alpha}}$ at the r.h.s. of (25c) one gets precisely (29b).

The comparison of the ϑ–equations is performed as follows. First we note that the matrix $\gamma^{(3)}$ can be written as

$$\gamma^{(3)} = iv_l h_{mn}\Gamma^{lmn}(1+\bar{\gamma}), \tag{41}$$

so that

$$
\begin{aligned}
1+\hat{\Gamma} &= (1+\gamma^{(3)})(1+\bar{\gamma}),\\
1-\hat{\Gamma} &= (1-\bar{\gamma})(1-\gamma^{(3)}).
\end{aligned}
\tag{42}
$$

By use of Eqs. (41), (42), the definition (18) and (35) it is a simple (but lengthy) exercise to prove the following matrix identities

$$
\begin{aligned}
\left(\frac{1+\bar{\Gamma}}{2}\right)\left(\frac{1+\hat{\Gamma}}{2}\right) &= \frac{1+\hat{\Gamma}}{2},\\
\left(\frac{1+\hat{\Gamma}}{2}\right)\left(\frac{1+\bar{\Gamma}}{2}\right) &= \frac{1+\bar{\Gamma}}{2}.
\end{aligned}
\tag{43}
$$

A convenient way to prove these matrix identites is to reduce them to identities between worldvolume tensors and then to verify the former in the particular $(\pm)$–basis as above. The relations (43) show that the κ–transformation of the two approaches are the same modulo a redefinition of the gauge parameter κ. Moreover, applying the first relation in (43) to the identity (25) one gets

$$
J^m(1+\hat{\Gamma}) = 0,
\tag{44}
$$

so that the field equation of ϑ (25a) in the action approach is written as

$$
E_m J^m (1-\hat{\Gamma}) = 0,
$$

or, using (42), as

$$
E_m J^m (1-\bar{\gamma}) = 0.
\tag{45}
$$

The last matrix identity required to complete the comparison is

$$
J^m(1-\bar{\gamma}) = -\frac{4}{1-\frac{2}{3}trk^2}\left(1-\gamma^{(3)}\right)\Gamma_n\,(1-\bar{\gamma})\,m^{mn}
\tag{46}
$$

To prove this identity one has to use, in particular, the relation

$$
T^{mn} - 2g^{mn} = -\frac{4m^{mn}}{1-\frac{2}{3}trk^2}.
$$

Therefore (45) coincides with (29a) apart from a (nonvanishing) overall scalar factor.

In conclusion we have shown that the equations of motion of the M–theory 5–brane obtained from the action principle are identical to the worldvolume component field equations derived from the doubly supersymmetric geometrical approach. The identification was established by solving the relation (28) between the auxiliary self–dual field h_{lmn} and the field strength H_{mnl} and expressing the former in terms of the latter (see Eq. (35)). To write this expression in a $d=6$ covariant way one had to use

the auxiliary scalar field $a(x)$. This resulted in linking the X and A_2 equations of these approaches. Then we found the γ–matrix identities (43) which allowed us to relate the parameters of the κ–transformations of the action and the geometrical approach and finally to identify the ϑ-equations.

A direction of further work might be studying the possibility of getting the complete set of the superfield equations of the M–theory five–brane [8] from a generalized action principle proposed in [13] for the description of superbranes in the doubly supersymmetric approach. This has already been done for $D = 10$ D-branes in [14].

One might also try to look for a 5–brane action which includes the field h_{mnl} instead of $a(x)$. Perhaps it would involve a covariant formulation of self–dual field dynamics (with infinite number of auxiliary (anti)–self–dual fields) developed in [18].

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